CSIR-UGC-NET/JRF | GATE

TIFR|BARC|JEST|IISc|DRDO|ISRO|SLET|ONGC-GT & Ph.D. Entrance

Assignment

Thermodynamics & Statistical PHysics

PHYSICAL SCIENCES

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PHYSICAL SCIENCES (JRF-NET/GATE)

Assignment-I: THERMODYNAMICS

1. Which of the following is not an exact differential?
   (a) $dQ(Q = \text{heat absorbed})$ (b) $dU(U = \text{internal energy})$
   (c) $dS(S = \text{entropy})$ (d) $dF(F = \text{free energy})$

2. If the entropy of a system remains constant in a thermodynamic process, the process is:
   (a) isobaric (b) isochoric (c) isothermal (d) adiabatic

3. The sole effect of a hypothetical thermodynamic process E_1 is to convert an amount Q of heat entirely into work. A second hypothetical process E_2 converts an amount W of work entirely into heat. It can be said that the second law of thermodynamics is violated by
   (a) E_1 but not E_2 (b) E_2 but not E_1 (c) both E_1 and E_2 (d) neither process.

4. Which of the following expresses the Second law of thermodynamics? (Symbols have their usual meanings).
   (a) $\Delta F \leq W$ (b) $\Delta S < Q/T$ (c) $\Delta U = Q + W$ (d) $\Delta S \leq 0$

5. A carnot engine operates between a heat source at 500 K and a heat sink at 300 K. The temperature of the source is increased by 20 K. In order that the efficiecy of the engine remain unchanged, the temperature of the sink should be changed by
   (a) + 20 K (b) +12 K (c) –12 K (d) –20 K

6. A carnot cycle operates on a working substance between two reservoirs at temperatures T_1 and T_2, with $T_1 > T_2$. During each cycle, an amount of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is **INCORRECT?**
   (a) Work done in one cycle is $Q_1 - Q_2$ (b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$
   (c) Entropy of the hotter reservoir decreases.
   (d) Entropy of the universe (consisting of the working substance and the two reservoir) increases.

7. If the isothermal compressibility of a solid is $\kappa_T = 10^{-10}\,(Pa)^{-1}$, the pressure required to increase its density by 1% is approximately.
   (a) $10^4\ Pa$ (b) $10^6\ Pa$ (c) $10^8\ Pa$ (d) $10^{10}\ Pa$

8. An ideal gas in a cylider is compressed adiabatically to one-third of its initial volume. During this process 20 J work is done on the gas by compressing agent. Which of the following statements is true for this case?
   (a) Change in the internal energy in this process is zero.
   (b) The internal energy increases by 20 J
   (c) The internal energy decreases by 20 J
   (d) Temperature of the gas decreases.

9. Mean total energy of a classical three-dimensional harmonical oscillator in equilibrium with a heat reservoir at temperature T is:
   (a) $k_\beta T$ (b) $\frac{3}{2}k_\beta T$ (c) $2\,k_\beta T$ (d) $3\,k_\beta T$

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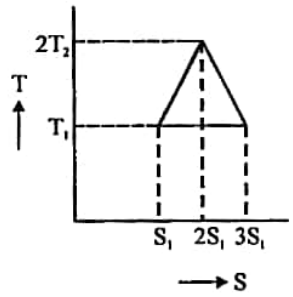
10. An amount of heat Q is transferred from a heat reservoir at temperature T_A to another heat reservoir at temperature T_B. What is the change in the entropy ΔS of the combined system?

(a) $Q\left(\frac{1}{T_B}-\frac{1}{T_A}\right)$ (b) $Q\left(\frac{1}{T_B}+\frac{1}{T_A}\right)$ (c) $\frac{Q}{\sqrt{T_B T_A}}\ln\frac{T_B}{T_A}$ (d) $Q\left(\frac{1}{T_A}-\frac{1}{T_B}\right)$

11. A reversible engine cycle is shown in the following T-S diagram. The efficiency of the engine is:



(a) 1/3 (b) 1/2 (c) 1/5 (d) 1/4

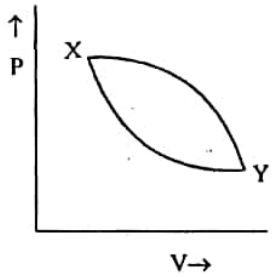
12. An engine absorbs heat at a temperature of 1000 K and rejects heat at 600 K. If the engine operates at maximum possible efficiency, the amount of work performed by the engine, for 2000 J heat input is:
(a) 1600 J (b) 1200 J (c) 800 J (d) 400 J

13. Boyle's law can be expressed in different form as
(a) $dV/dP = 1$ (b) $dV/dP = V/P$ (c) $dV/dP = P/V$ (d) $dV/dP = -V/P$

14. For any process, the second law of thermodynamics requires that the change of entropy of the universe be
(a) positive only (b) positive or zero (c) zero only (d) negative or zero

15. A sample of ideal gas with initial pressure P and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constant is R. Then the work done by the gas is given by
(a) $(PV\Delta S)/(nR)$ (b) $nR\Delta S$ (c) PV (d) $(P\Delta S)/(nRV)$

16. A piston containing an ideal gas is originally in the state X (see figure). The gas is taken through a thermal cycle X→Y→X as shown:
The work done by the gas is positive if the direction of the thermal cycle is:
(a) clockwise
(b) counter - clockwise
(c) neither clockwise nor counter-clockwise
(d) clockwise from X→Y and counter-clockwise from Y→X.



17. Hydrogen molecules (mass m) are in thermal equilibrium at a temperature T. Assuming classical distribution of velocity, the most probable speed at room temperature is:
(a) $(k_B T)/m$ (b) $2k_B T/m$
(c) $\left(\sqrt{2}k_B T/m\right)$ (d) $mn/\left(\sqrt{2}k_B T\right)$

18. The root-mean square speed of a particle of mass m in the kinetic theory is given by
(a) $\sqrt{\frac{k_\beta}{m}}$ (b) $\sqrt{\frac{2k_\beta}{m}}$ (c) $\sqrt{\frac{3k_\beta}{m}}$ (d) $\sqrt{\frac{8k_\beta}{m}}$

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19. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is:

(a) $pV^{1/3} = \text{constant}$ (b) $pV^{2/3} = \text{constant}$ (c) $pV^{4/3} = \text{constant}$ (d) $pV^{5/3} = \text{constant}$

20. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is:

(a) $\frac{1}{2}k_BT$ (b) k_BT (c) $\frac{3}{2}k_BT$ (d) $3k_BT$

21. A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22°C by suppling 450 kJS^{-1}. If the outside temperature is 0°C, the heat taken, in kJs^{-1}, from the outside air is approximately.
(a) 487 (b) 470 (c) 467 (d) 417

22. The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by in $p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in Kelvin) of the material is:
(a) 185 (b) 190 (c) 195 (d) 200

23. The internal energy of n moles of a gas is given $E = \frac{3}{2}nRT - \frac{a}{V}$, where V is the volume of the gas at temperature T and *a* is a positive constant. One mole of the gas is state (T_1, V_1) is allowed to expand adiabatically into vacuum to a final state (T_2, V_2). The temperature T_2 is:

(a) $T_1 + Ra\left(\frac{1}{V_2} + \frac{1}{V_1}\right)$ (b) $T_1 - \frac{2}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

(c) $T_1 + \frac{2}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$ (d) $T_1 - \frac{1}{3}Ra\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$

24. The pressure versus temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid-to-solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is:
(a) the entropy of solid is zero in this temperature region.
(b) the entropy increases when the system goes from liquid to solid phase in this temperature region.
(c) the entropy decreases when the system transforms from liquid to solid phase in this region of temperature.
(d) the change in entropy is zero in the liquid-to-solid transition region.

25. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium,
(a) the total entropy of the final system is the same as the sum of the entropy of the individual system always.
(b) the entropy of mixing is $2Nk_B \ln 2$.
(c) the entropy of the final system is less than that of sum of the initial entropies of the two gases.
(d) the entropy of mixing is non-zero when the atoms A and B are of the same type.

26. The mean free path of the particles of a gas at temperature T_0 and pressure p_0 has a value λ_0. If the pressure is increased to $1.5\,p_0$ and the temperature is reduced to $0.75\,T_0$, the mean free path
(a) remains unchanged. (b) is reduced to half
(c) is doubled (d) is equal to $1.125\,\lambda_0$

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27. The boiling point of a liquid at pressure P_0 is T_0. Its molar latent heat of vaporisation is L and molar volume of the liquid phase is negligible as compared to the vapour phase. The vapour phase obeys perfect gas equation. The boiling point T at pressure P is given by

(a) $P/P_0 = (L/RT_0)(1-T_0/T)$ (b) $\ln(P/P_0) = (L/RT_0)(1-T_0/T)$

(c) $\ln(P/P_0) = (L/RT_0)(1-T/T_0)$ (d) $P/P_0 = 1+(L/RT_0)\ln(T_0/T)$

28. The ratio $\gamma = C_P/C_V$ of H_2 gas is 7/5 at room temperature, and 5/3 at low temperatures (around 50 K). The reason for this change is

(a) Dissociation of H_2 molecules (b) Suppression of rotational degrees of freedom

(c) Formation fo random clusters of molecules. (d) Liquefaction of the gas.

29. For an isolated thermodynamical system p, V, T, U, S, and F represent the pressure, volume, temperature, internal energy, entropy, and free energy respectively. Then the following relation is true

(a) $\left(\frac{\partial F}{\partial T}\right)_V = -S$ (b) $\left(\frac{\partial F}{\partial T}\right)_P = -S$ (c) $\left(\frac{\partial U}{\partial T}\right)_P = T$ (d) $\left(\frac{\partial U}{\partial V}\right)_T = P$

30. A quantity of water is completely converted into steam by boiling.

(a) This is a second order phase transition.

(b) At the boiling point, the chemical potential in the vapour phase is less than that in the liquid phase.

(c) The boiling temperature would increse if the boiling were done under increased pressure.

(d) The entropy of the steam is greater than that of the water.

31. In a first order phase transition, at the transition temperature, specific heat of the system

(a) Diverges and its entropy remains the same

(b) Diverges and its entropy has finite discontinuity.

(c) Remains unchanged and its entropy has finite discontinuity.

(d) Has finite discontinuity and its entropy diverges.

32. Consider the transition of liquid water to steam as water boils at a temperature of 100°C under a pressure of 1 atmosphere. Which one of the following quantities does not change discontinuously at the transition?

(a) The Gibbs free energy (b) The internal energy

(c) The entropy (d) The specific volume

33. The sublimation curve of solid ammonia is given by ln p=23–3750/T and the vaporization curve of the liquid ammonia is given by ln p=19.5–3050/T, where p is in mm of Hg and T is in °K. The temperature of the triple point of ammonia is:

(a) 3750 K (b) 3050 K (c) 700 K (d) 200 K

34. Which of the following is an example of a first order phase transition?

(a) A liquid - gas phase transition at the critical point.

(b) A paramagnet - ferromagnet phase transition

(c) A normal metal - superconductor phase transition.

(d) A liquid - gas phase transition away from the critical point.

35. A second order phase transition is one in which

(a) the plot of entropy as a function of temperature shows a discontinuity.

(b) the plot of specific heat as a function of temperature shows a discontinuity

(c) the plot of volume as a function of pressure shows a discontinuity.

(d) the plot of comprehensibility as a function of temperature is continuous.

36. Identify which one is a first order phase transition?

(a) A liquid to gas transition at its critical temperature.

(b) A liquid to gas transition close to its triple point.

(c) A paramegnetic to ferromagnetic transition in the absence of a magnetic field.

(d) A metal to superconductor transition in the absence of a magnetic field.

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37. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be (k is the Boltzmann constant and T is non zero)

(a) Zero (b) Infinite (c) $\frac{3}{2}kT$ (d) kT

only for non relativistic particle

$h_B \neq 0$ Quantization then classically

Answer Key (THERMODYNAMICS-I)

1. (a)	2. (d)	3. (a)	4. (b)	5. (b)	6. (d)	7. (c)
8. (b)	9. (d)	10. (a)	11. (a)	12. (c)	13. (d)	14. (a)
15. (a)	16. (a)	17. (c)	18. (c)	19. (c)	20. (b)	21. (d)
22. (d)	23. (c)	24. (d)	25. (b)	26. (b)	27. (b)	28. (a)
29. (a)	30. (d)	31. (b)	32. (a)	33. (d)	34. (d)	35. (b)
36. (b)	37. (d)					

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PHYSICAL SCIENCES (JRF-NET/GATE)

Assignment-II: THERMODYNAMICS

1. A thermodynamic system is classified as closed if it can
   (a) Exchange energy with its surroundings, but not matter.
   (b) Exchange both energy and matter with its surroundings.
   (c) Exchange neither energy nor matter with its surroundings
   (d) Exchange only matter, but not energy, with its surroundings.

2. The first law of thermodynamics states that $\Delta U = \Delta Q - \Delta W$, where the symbols have their usual meaning. Which one of the following quantity(ies) depend(s) only on the initial and the final state, and is (are) independent of the path of the system between these states?
   (a) ΔQ (b) ΔW (c) ΔU (d) ΔU and ΔW

3. In an infinitesimal reversible thermodynamic process, which of the following quantity depends only on the initial and the final states of the system and is independent of the path between these states?
   (a) $T\Delta Q$ (b) $\Delta Q/T$ (c) $T/\Delta Q$ (d) $T^2/\Delta Q$

4. Which of the following statements is correct?
   (a) You can construct an engine operating in cycles, extracting heat Q and performing exactly equivalent work W.
   (b) You can construct an engine operating in cycles extracting heat Q and performing more than equivalent amount of work W.
   (c) You can have an engine operating cyclically such that the entropy remains constant throughout the cycle.
   (d) You cannot construct a cyclically operating engine that will extract heat Q and perform equivalent amount of work W.

5. Above a temperature T_0, the specific heat of a certain gas is approximate 3.5R, and is approximately 2.5R below T_0. The gas is most likely
   (a) oxygen (b) ozone (c) carbon dioxide (d) argon

6. During the isothermal reversible expansion of an ideal gas,
   (a) the entropy and the pressure both increase
   (b) the entropy and the pressure both decrease
   (c) the entropy increases and the pressure decreases
   (d) the entropy decreases and the pressure increases

7. The isothermal compressibility, κ of an ideal gas at temperature T_0 and volume V_0, is given by
   (a) $-\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial P}\Big|_{T_0}$ (c) $-V_0\frac{\partial P}{\partial V}\Big|_{T_0}$ (d) $V_0\frac{\partial P}{\partial P}\Big|_{T_0}$

8. Let ΔW be the work done in a quasistatic reversible thermodynamics process. Which of the following statements about ΔW is correct?
   (a) ΔW is a perfect differential if the process is isothermal
   (b) ΔW is a perfect differential if the process is adiabatic
   (c) ΔW is always a perfect differential
   (d) ΔW cannot be a perfect differential.

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9. An adiabat in the PV-plane is a curve of
(a) Constant temperature (b) Constant entropy
(c) Costant pressure (d) Constant volume

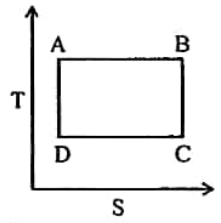
10. The volume of a thermodynamic system increases irreversibly by an incremental amount δV. If P is the pressure, the work done on the system is:
(a) $\delta W = P\,\delta V$ (b) $\delta W = -P\,\delta V$ (c) $\delta W < -P\,\delta V$ (d) $\delta W > -P\,\delta V$

11. Consider an elastic string of length L under tension τ at temperature T. Let U and S be its internal energy and entropy, respectively. Then the conjugate pairs of thermodynamic variables are
(a) (τ, T) and (L, S) (b) (L, τ) and (T, S)
(c) (T, L) and (τ, U) (d) (L, S) and (T, U)

12. A carnot engine can be used to define an absolute temperature scale. This is made possible by the following property of the carnot engine.
(a) It involves an adiabatic as well as an isothermal process.
(b) It is a cyclic process.
(c) It is most efficient engine operating between two given temperatures.
(d) It conserves energy.

13. Which is the most effective way of increasing the efficiency of a carnot engine operating between temperatures T_1 and $T_2 (T_1 > T_2)$?
(a) Increasing T_1, keeping T_2 fixed. (b) Increasing T_2, keeping T_1 fixed.
(c) Decreasing T_2 and increasing T_1 (d) Increasing T_2 and decreasing T_1.

14. Entropy - temperature (S-T) diagram of a Carnot cycle is shown below:



The adiabatic compression is represented by
(a) AB (b) DA (c) CD (d) BC

15. The quantity that remains constant in Joule-Thomson expansion is
(a) $U + pV$ (b) $U + pV - TS$ (c) U (d) $U - TS$

16. A steam turbine is operated with an intake temperature 407°C and exhaust temperature 117°C. Heat input to the turbine is Q. Then maximum work turbine can do is:
(a) 0.5Q (b) 0.37Q (c) Q (d) 0.25 Q

17. For a system with constant volume and temperature, the thrmodynamic equilibrium corresponds to the state with minimum?
(a) Helmholtz free energy (b) Gibbs thermodynamic potential
(c) Internal energy (d) Enthalpy

18. A gas of argon atoms is kept in a container of volume V and pressure P and is in equilibrium with a heat bath at temperature T. The average energy of an argon atom in the gas is:
(a) RT (b) $3k_nT$ (c) $3/2\,k_BT$ (d) 3 RT

Where k_B is Boltzman constant.

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19. A Carnot cycle operates on a working substance between two reservoirs at temperature T_1 and T_2, with $T_1 > T_2$. During each cycle, an amount of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is INCORRECT?

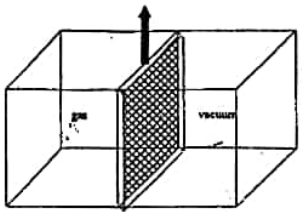
(a) Work done in one cycle is $Q_1 - Q_2$.

(b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_1}$

(c) Entropy of the hotter reservoir decreases

(d) Entropy of the universe (cosisting of the working substance and the two reservoirs) increases.

20. Consider a sealed but thermally conducting container of total volume V, Which is in equilibrium with athermal bath at temperature T. The container is divided into two equal chambers by a thin but impermeable partition. One of these chambers contains an ideal gas, while the other half is a vacuum (see figure)



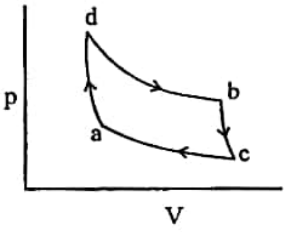
If the partition is removed and the ideal gas is allowed to expand and fill the entire container, then the entropy per molecule of the system will increase by an amount

(a) $2k_B$ (b) $k_B \ell n(1/2)$ (c) $k_B \ell n2$ (d) $(k_B \ell n2)/2$

21. The internal energy E of a system is given by $E = \frac{bS^3}{VN}$, where b is a constant and other symbols have their usual meaning . The temperature of this system is equal to

(a) $\frac{bS^2}{VN}$ (b) $\frac{3bS^2}{VN}$ (c) $\frac{bS^3}{V^3N}$ (d) $\left(\frac{S}{N}\right)^2$

22. In the Carnot cycle shown in the figure, the curves



(a) ab and cd are adiabatic, bc and da are isothermal

(b) ab and cd are isothermal, bc and da are adiabatic

(c) ab and cd are isothermal, bc and da are isobaric

(d) ab and cd are isobaric, bc and da are isothermal.

23. An ideal gas engine produces 1 kJ of useful work. If it operators between 400K and 300K, the amount of energy lost as waste heat is

(a) 4 kJ (b) 2kJ (c) 1kJ (d) 3kJ

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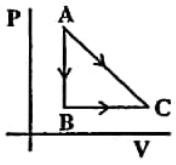
24. A monoatomic ideal gas at 17°C is adiabatically compressed to 1/8 of its original volume. The temperature after compression is

(a) 2.1°C (b) 17°C (c) –200.5°C (d) 887° C

25. Efficiency of perfectly reversible (Carnot) heat engine operation between absolute temperature T and zero is equal to

(a) 0 (b) 0.5 (c) .75 (d) 1

26. A given quantity of gas is taken from the state $A \rightarrow C$ reversibly, by two paths, $A \rightarrow C$ directly and $A \rightarrow B \rightarrow C$ as shown in the figure below.



During the $A \rightarrow C$ the work done by the gas is 100 J and the heat absorbed is 150 J. If during the process $A \rightarrow B \rightarrow C$ the work done by the gas is 30 J, the heat absorbed is:

(a) 20J (b) 80J (c) 220 J (d) 280 J

27. The internal energy U is related to the Helmholtz free energy F by the relation:

(a) $U = -T\left\{\frac{\partial^2}{\partial T^2}\left(\frac{F}{T}\right)\right\}_v$ (b) $U = T\left\{\frac{\partial^2}{\partial T^2}\left(\frac{F}{T}\right)\right\}_v$

(c) $U = -T^2\left\{\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right\}_v$ (d) $U = T^2\left\{\frac{\partial}{\partial T}\left(\frac{F}{T}\right)\right\}_v$

28. Two insulated containers connected with a stop cock are initially evaluated. An ideal gas is filled in one of them and then allowed to expand into the other container by opening the cock. In this process the work done (W), the heat evolved (Q) and the change in internal energy (ΔU) are given by

(a) $W = 0, Q = 0$ and $\Delta U = 0$ (b) $W = +ve, Q = 0$ and $\Delta U = -ve$

(c) $W = -ve, Q = 0$ and $\Delta U = -ve$ (d) $W = -ve, Q = 0$ and $\Delta U = +ve$

29. A simple calculation for the entropy S of an ideal gas gives $S = \frac{3}{2} N k_B \ln V + S_0$ where N is the number of molecules in the gas, V is its volume, and S_0 an arbitrary constant. This expression is erroneous because

(a) It does not contain temperature T
(b) It does not contain pressure P
(c) It is not extensive
(d) It does not take into account the interaction between the molecules of the gas.

30. Imagine dividing a fluid with internal energy V, entropy S, volume V, particle number N, Temperature T, pressure P and chemical potential μ into two equal parts. The variables that remain unaffected by this division are

(a) U, V, S and N (b) U, V, T and P (c) T, P and μ (d) T, P and S

31. N_1 molecules of oxygen are confined in volume V_1 at temperature T and have entropy S_1; N_2 molecules of oxygen are confined in volume V_2 at temperature T and have entropy S_2. The gases are mixed in a volume $V = V_1 + V_2$. The entropy S of the mixture is:

(a) $S < S_1 + S_2$ (b) $S = S_1 + S_2$ (c) $S > S_1 + S_2$ (d) $RT\left(\ln\frac{V_1}{V_1+V_2} - \ln\frac{V_2}{V_1+V_2}\right)$

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32. Which of the following expressions are correct?
(a) $dH = TdS + VdP$, Where H is enthalpy
(b) $dF = SdT + PdV$, Where F is Helmholtz free energy.
(c) $dG = SdT - VdP$, Where G is Gibbs function
(d) $SdT = dE + PdV$, where E is internal energy.

33. Consider a perfect gas in a vessel rushing into an evacuated vessel and let the whole system be isolated. Let V_1 and V_2 denote the initial and final volumes of the gas. The increase in entropy of the system is:
(a) Zero (b) $R \ln \frac{V_2}{V_1}$ (c) $\frac{1}{T}\int_{V_1}^{V_2} du + R \ln \frac{V_2}{V_1}$ (d) $\frac{1}{T}\int_{V_1}^{V_2} du$

34. In order to attain thermal equilibrium at a fixed volume, the system minimises,
(a) S (b) $H = E + pV$ (c) $G = U - TS + pV$ (d) $F = U - TS$
Where the symbols have their usual meaning.

35. The particle density fluctuations in a pure substance are maximum at
(a) Its freezing point (b) Its boiling point (c) At critical point (d) At triple point.

36. A thermally insulated cubical box has two chambers of equal volume. Initially, one mole of a mono atomic ideal gas is placed in one of the chambers while the other chamber is kept empty. The gas slowly leaks through a small hole and eventually occupies the whole box. The change in entropy of the gas is given by
(a) $R/\ln 2$ (b) $R \ln 2$ (c) zero (d) 2R

37. One mole of ideal monoatomic gas is kept in a volume V and is at temperature T. If the volume and temperature of the gas are respectively changed to V' and T'. in such a way that the entropy of the gas is unchagned, then which of the following is true?
(a) $V' = 2V$, $T' = T/2$ (b) $V' = 8V$, $T' = T/4$
(c) $V' = 4V$, $T' = 8T$ (d) $V' = V/2$, $T' = 2T$

38. Gas molecules of mass 'm' are confined in a cylinder of radius R and height L (with $R \gg L$) kept vertically in the Earth's gravitational field. The average energy of the gas at low temperatures (such that $mgL >> k_B T$) is given by
(a) $Nk_BT/2$ (b) $3Nk_BT/2$ (c) $2Nk_BT$ (d) $5Nk_BT/2$

39. Consider a thermodynamic system of N particles with internal energy U, volume V, and entropy S. Its chemical potential is given by
(a) $\left(\frac{\partial U}{\partial S}\right)_{V,N}$ (b) $\left(\frac{\partial N}{\partial U}\right)_{S,V}$ (c) $\left(\frac{\partial U}{\partial V}\right)_{S,N}$ (d) $\left(\frac{\partial U}{\partial N}\right)_{S,V}$

40. The heat capacity at costant pressure of a certain gas is 1.4 times its heat capacity at constant volume. A best model for the molecules of this gas is
(a) rigid triangles (b) point particles
(c) rigid dumbbells (d) non- rigid dumbbells that can vibrate

41. A carnot engine operates between a source at temperature 300K and a sink at temperature 200 K. Let η be the efficiency of the Carnot engine when the temperatures of both the source and the sink are decreased by the same amount. Then,
(a) $\eta < 1/3$ (b) $\eta > 1/3$ (c) $\eta = 1/3$ (d) $\eta = 1/2$

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42. Consider the Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial P}{\partial T}\right)_{V,N}$ where the symbols have the usual meaning. this relation can be derived from

(a) Helmholtz free energy. $F = U - TS$

(b) Gibbs free energy : $G = U - TS + PV$

(c) Grand thermodynamic potential $Z = U - TS - \mu N$

(d) Enthaply $H = U + PV$

43. We have $S \equiv (U, V, N)$, S = entropy, V= volume, U= internal energy, N= number of particles, P = Pressure, T = Temperature, μ = chemical Potential. Which of the following statements is correct?

(a) $\left(\frac{\partial S}{\partial V}\right)_{U,N} = -\frac{1}{T}$ (b) $\left(\frac{\partial S}{\partial V}\right)_{U,N} = -\frac{P}{T}$ (c) $\left(\frac{\partial S}{\partial V}\right)_{U,N} = \frac{P}{T}$ (d) $\left(\frac{\partial S}{\partial N}\right)_{V,U} = \frac{\mu}{T}$

44. Consider an ideal gas of mass m at temperature T_1 which is mixed isobarically (i.e., at constant pressure) with an equal, mass of same gas at temperature T_2 in a thermally insulated container. What is the change of entropy of the universe?

(a) $2mC_p \ln\left(\frac{T_1 + T_2}{2\sqrt{T_1 T_2}}\right)$ (b) $2mC_p \ln\left(\frac{T_1 - T_2}{2\sqrt{T_1 T_2}}\right)$

(c) $2mC_p \ln\left(\frac{T_1 + T_2}{2T_1 T_2}\right)$ (d) $2mC_p \ln\left(\frac{T_1 - T_2}{2\sqrt{T_1 T_2}}\right)$

45. A thermally insulated ideal gas of volume V_1 and temperature T expends into another enclosure of volume V_2 through a porous plug. What is the change in the temperature of the gas ?

(a) 0 (b) $T \ln(V_1 / V_2)$ (c) $T \ln(V_2 / V_1)$ (d) $T \ln[(V_2 - V_1)/V_2]$

46. The free energy of a gas N particled in a volume V and at a temperature T is $F = Nk_B T \ln\left[a_0 V (k_B T)^{5/2} / N\right]$, where a_0 is a constant and k_B denotes the Boltzmann constant. The internal energy of gas is

(a) $\frac{3}{2} Nk_B T$ (b) $\frac{5}{2} Nk_B T$

(c) $Nk_B T \ln\left[a_0 V (k_B T)^{5/2} / N\right] - \frac{3}{2} Nk_B T$ (d) $Nk_B T \ln\left[a_0 V / (k_B T)^{5/2}\right]$

47. Consider a Maxwellian distribution of the velocity of the molecules of an ideal gas. Let V_{mp} and V_{rms} denote the most probable velocity and the root mean square velocity, respectively. The magnitude of the ratio V_{mp} / V_{rms} is

(a) 1 (b) $2/3$ (c) $\sqrt{2/3}$ (d) $3/2$

48. Consider a sample of argon gas contained in a box of volume V. The root-mean square speed of the argon atoms is v_m. If the gas is allowed to expand at constant pressure to a volume of 2v, the root mean square speed of the gas molecules would be

(a) $\upsilon_m/2$ (b) $\sqrt{2\upsilon_m}$ (c) $\upsilon_m/\sqrt{2}$ (d) $2\upsilon_m$

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49. A classical ideal gas in an enclosure is in equilibrium with a heat bath at a constant temperature. The ratio of the root-mean square speed V_{rms}, mean speed υ_{mean} and the most probable speed υ_{mp} namely $V_{rms} : V_{mean} : V_{mp}$ is:

(a) $\sqrt{3} : \sqrt{\frac{8}{\pi}} : \sqrt{2}$ (b) $\frac{\sqrt{3}}{2} : \sqrt{\frac{2}{\pi}} : 1$ (c) $\sqrt{\pi} : \sqrt{3} : \sqrt{2}$ (d) $\sqrt{\frac{\pi}{3}} : \sqrt{\frac{\pi}{8}} : \frac{1}{\sqrt{2}}$

50. The average speed of molecules in a gas at 20°C is V. At what temperature it will be 2V?

(a) 899°C (b) 586°C (c) 80°C (d) 40°C

51. A classical gas of molecules each of mass m, is in thermal equilibrium at the absolute temperature, T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $(v_x + v_y)^2$ is:

(a) $\frac{k_B T}{m}$ (b) $\frac{3}{2}\frac{k_B T}{m}$ (c) $\frac{1}{2}\frac{k_B T}{m}$ (d) $\frac{2k_B T}{m}$

52. At a certain temperature T, the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are, respectively,

(a) 355 m/s, 434 m/s (b) 820 m/s, 917 m/s
(c) 152 m/s, 301 m/s (d) 422 m/s, 600 m/s

53. At 100°C, water vapour and liquid water coexist in thermodyanics equilibrium in a closed container. If μ and S represent the chemical potential and entropy respectively, then at the interface.

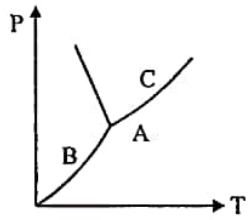
(a) $\mu_{vapour} = \mu_{liquid}$ (b) $S_{vapour} = S_{liquid}$
(c) $S_{vapour} < S_{liquid}$ (d) $\mu_{vapour} > \mu_{liquid}$

54. Finite changes in a second order phase transition occur in

(a) S (b) V (c) C_p (d) T

Where the symbols have their standard meaning.

55. The pressure versus temperature diagram for a pure substance is shown below.



The region represented by A, B & C respectively are

(a) Solid, liquid and vapour (b) Vapour, solid and liquid
(c) Solid, vapour and liquid (d) Vapour liquid and solid.

56. The transition from ordinary liquid helium (He I) to superfluid helium (He II) is an example of a second order lambda point transition. The quantities that remain constant in such a transition at the transition point are

(a) entropy, volume, specific heat at constant pressure and Gibbs free energy
(b) pressure, volume and isothermal compressibility.
(c) specific heat at constant pressure, isothermal compressibility and pressure.
(d) pressure, volume, entropy and Gibbs free energy.

57. In a first order phase transition, at the transition temperature, specific heat of the system.

(a) Diverges and its entropy remains the same
(b) Diverges and its entropy has finite discontinuity
(c) Remains unchanged and its entropy has finite discontinuity.
(d) Has finite discontinuity and its entropy diverges.

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58. When two phases 1 and 2 of a system are in thermodynamic equilibrium, their temperature T, pressure p and chemical potential μ satisfy:

(a) $T_1 = T_2$, $p_1 = p_2$ and $\mu_1 = \mu_2$ (b) $T_1 = T_2$, $p_1 = p_2$ and $\mu_1 \neq \mu_2$

(c) $T_1 = T_2$, $\mu_1 = \mu_2$ and $p_1 \neq p_2$ (d) $T_1 = T_2$ but $p_1 \neq p_2$, $\mu_1 \neq \mu_2$

59. When a crystal melts into a liquid at constant pressure, which of the following quantities changes continuously at the phase transition?

(a) Gibbs free energy (b) Volume (c) Internal energy (d) Entropy

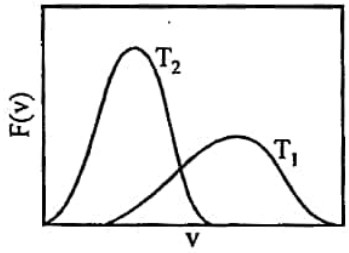
60. Consider a gas of atoms obeying Maxwell-Boltzman statistics. The average value of $e^{i\vec{a}.\vec{p}}$ over all the momenta $\vec{p}$ of each of the particles (where $\vec{a}$ is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is,

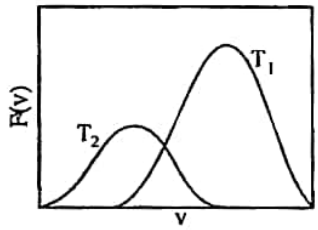
(a) One (b) Zero (c) $e^{\frac{1}{2}a^2mkT}$ (d) $e^{-\frac{3}{2}a^2mkT}$

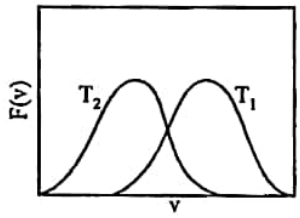
61. Ten grams of ice at 0°C is added to a beaker containing 30 grams of water at 25°C. What is the final temperature of the system when it comes to thermal equilibrium? (The specific heat of water is 1cal/gm/°C and latent heat of melting of ice is 80 cal/gm)

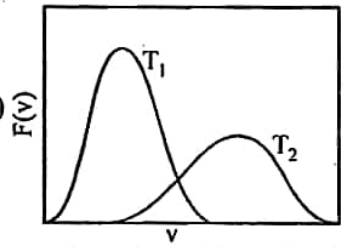
(a) 0°C (b) 7.5°C (c) 12.5°C (d) −1.25°C

62. For temperature $T_1 > T_2$, the qualitative temperature dependence of the probability distribution F(v) of the speed v of a molecule in three dimensions is correctly represented by the following figure:

(a)


(b)


(c)


(d)


63. The speed v of the molecules of mass m of an ideal gas obeys Maxwell's velocity distribution law at an equilibrium temperature T. Let (v_x, v_y, v_z) denote the components of the velocity and k_B the Boltzmann constant. The average value of $(\alpha v_x - \beta v_y)^2$, where α and β are constants, is

(a) $(\alpha^2 - \beta^2)k_\beta T/m$ (b) $(\alpha^2 + \beta^2)k_\beta T/m$ (c) $(\alpha+\beta)^2 k_\beta T/m$ (d) $(\alpha-\beta)^2 k_\beta T/m$

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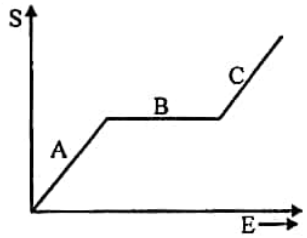
64. The entropy S of a thermodynamic system as a function of energy E is given by the following graph



The temperatures of the phases A, B and C, denoted by T_A, T_B and T_C, respectively, satisfy the following inequalities:

(a) $T_C > T_B > T_A$ (b) $T_A > T_C > T_B$ (c) $T_B > T_C > T_A$ (d) $T_B > T_A > T_C$

65. A Carnot cycle operates as a heat engine between two bodies of equal heat capacity until their temperatures become equal. If the initial temperatures of the bodies are T_1 and T_2, respectively, and $T_1 > T_2$ then their common final temperature is

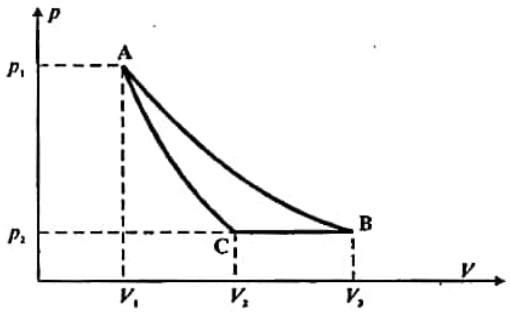
(a) T_1^2 / T_2 (b) T_2^2 / T_1 (c) $\sqrt{T_1 T_2}$ (d) $\frac{1}{2}(T_1 + T_2)$

66. Two different thermodynamic systems are described by the following equations of state:

$\frac{1}{T^{(1)}} = \frac{3RN^{(1)}}{2U^{(1)}}$ and $\frac{1}{T^{(2)}} = \frac{5RN^{(2)}}{2U^{(2)}}$ where $T^{(1,2)}, N^{(1,2)}$ and $U^{(1,2)}$ are respectively, the temperatures, the mole numbers and the internal energies of the two systems, and R is the gas constant. Let U_{tot} denote the total energy when these two systems are put in contact and attain thermal equilibrium. The ratio $\frac{U^{(1)}}{U_{tot}}$ is

(a) $\frac{5N^{(2)}}{3N^{(1)} + 5N^{(2)}}$ (b) $\frac{3N^{(1)}}{3N^{(1)} + 5N^{(2)}}$ (c) $\frac{N^{(1)}}{N^{(1)} + N^{(2)}}$ (d) $\frac{N^{(2)}}{N^{(1)} + N^{(2)}}$

67. One mole of an ideal gas undergoes the cycle ACBA shown in the PV diagram below.



One of the curved lines in the cycle represents an isothermal change at temperature T, while the other represents an adiabatic change. The net heat gained by the gas in this cycle is

(a) $-p_2(V_3 - V_2) + RT \ln \frac{V_2}{V_1}$ (b) $-p_2(V_3 - V_2) + RT \ln \frac{V_3}{V_1}$

(c) $-p_2(V_3 - V_2) + \gamma RT\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$ (d) $(p_1V_1 - p_2V_2) - RT \ln \frac{V_3}{V_1}$

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68. An ideal gas at a temperature T is enclosed in a rigid container whose walls are initially at temperature T_1, where $T_1 < T$. The walls are covered on the outside with perfect thermal insulation and the system is allowed to come to equilibrium. The pressure exerted by the gas on the walls of the container

(a) remains constant throughout

(b) is lower at the initial state than at the final state.

(c) is higher at the initial stage than at the final state.

(d) is the same at the initial and final stages.

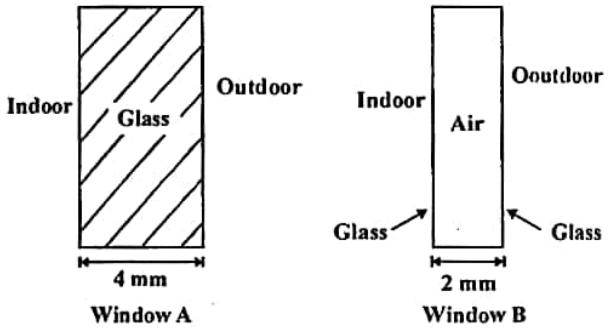
69. A thermally-insulated container of volume V_0 is divided into two equal halves by a non-permeable partition. A real gas with equation of state.

$$b^3\left(p+\frac{a^2}{V^3}\right)=nRT$$

where 'a' and 'b' are constants, is confined to one of these halves at a temperature T_0. The partition is now removed suddenly and the gas is allowed to expand to fill the entire container. The final temperature of the gas, in terms of its specific heat C_V, will be

(a) $T_0-\frac{3a^2}{2C_VV_0^2}$ (b) $T_0-\frac{2a^2}{3C_VV_0^2}$ (c) $T_0+\frac{3a^2}{2C_VV_0^2}$ (d) $T_0+\frac{2a^2}{3C_VV_0^2}$

70. A manufacturer is able to offer two models of heat-conserving windows, as described below



Window A is a simple pane of glass, 4 mm thick. Window B, on the other hand, consists of two extremely thin panes of glass, separated by an air gap of 2mm, as shown in the figure below above. If the thermal conductivity of glass is known to be 0.8 $Wm^{-1}K^{-1}$ and that of air is 0.025 $Wm^{-1}K^{-1}$, then the ratio of heat flow Q_A through window A to the heat flow Q_B through Window B is given by

$$\frac{Q_A}{Q_B}=$$

(a) $\frac{1}{16}$ (b) $\frac{1}{4}$ (c) 4 (d) 16

71. The free energy F of a system depends on a thermodynamics variable ψ as

$$F=-\alpha\psi^2+b\psi^6$$

with a, b > 0. The value of ψ, when the system is in thermodynamic equilibrium, is

(a) zero (b) $\pm(a/6b)^{1/4}$ (c) $\pm(a/3b)^{1/4}$ (d) $\pm(a/b)^{1/4}$

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Answer Key (THERMODYNAMICS-II)

1. (c)	2. (c)	3. (b)	4. (d)	5. (a)	6. (c)	7. (a)
8. (b)	9. (b)	10. (b)	11. (a)	12. (c)	13. (c)	14. (b)
15. (a)	16. (a)	17. (a)	18. (c)	19. (d)	20. (c)	21. (b)
22. (b)	23. (d)	24. (d)	25. (d)	26. (b)	27. (c)	28. (a)
29. (c)	30. (c)	31. (c)	32. (a)	33. (b)	34. (d)	35. (c)
36. (b)	37. (b)	38. (d)	39. (d)	40. (c)	41. (b)	42. (a)
43. (c)	44. (a)	45. (a)	46. (d)	47. (c)	48. (b)	49. (a)
50. (a)	51. (d)	52. (b)	53. (a)	54. (c)	55. (b)	56. (d)
57. (b)	58. (a)	59. (a)	60. (c)	61. (a)	62. (a)	63. (b)
64. (c)	65. (c)	66. (b)	67. (b)	68. (c)	69. (a)	70. (d)
71. (c)						

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CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
Institute for NET-JRF/GATE & IIT-JAM Exams

PHYSICAL SCIENCES (NET-JRF/GATE)

Assignment-I: STATISTICAL

1. A microcanonical ensemble represents
(a) a system in contact with a heat reservoir
(b) an isolated system in equilibrium
(c) a system that can exchange particles with its surroundings.
(d) a system under constant external pressure.

Consider a two level quantum system with energies $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon$

2. The Helmholtz free energy of the system is given by
(a) $-k_B T \ln\left(1+e^{-\epsilon/k_B T}\right)$ (b) $k_B T \ln\left(1+e^{-\epsilon/k_B T}\right)$
(c) $\frac{3}{2}k_B T$ (d) $\epsilon - k_B T$

3. The specific heat of the system is given
(a) $\frac{\epsilon}{k_B T}\frac{e^{-\epsilon/k_B T}}{\left(1+e^{-\epsilon/k_B T}\right)^2}$ (b) $\frac{\epsilon^2}{k_B T}\frac{e^{-\epsilon/k_B T}}{\left(1+e^{-\epsilon/k_B T}\right)}$ (c) $\frac{\epsilon^2}{}\frac{e^{-\epsilon/k_B T}}{\left(1+e^{-\epsilon/k_B T}\right)^2}$ (d) $\frac{\epsilon^2}{k_B T}\frac{e^{-\epsilon/k_B T}}{\left(1+e^{-\epsilon/k_B T}\right)^2}$

4. The probability that an energy level ϵ at a temperature T is unoccupied by a fermion of chemical potential μ is given by
(a) $\frac{1}{e^{(\epsilon-\mu)/k_B T}+1}$ (b) $\frac{1}{e^{(\epsilon-\mu)/k_B T}-1}$ (c) $\frac{1}{e^{(\mu-\epsilon)/k_B T}+1}$ (d) $\frac{1}{e^{(\mu-\epsilon)/k_B T}-1}$

Common data for Q5 and Q6.
Consider two dimentional electron gas of N electrons of mass 'm' each in a system of size L×L.

5. The density of states between energy ϵ and $\epsilon+d\epsilon$ is
(a) $\frac{4\pi L^2 m}{h^2}$ (b) $\frac{4\pi L^2 m}{h^2}\frac{1}{\sqrt{\epsilon}}$ (c) $\frac{4\pi L^2 m}{h^2}\sqrt{\epsilon}$ (d) $\frac{4\pi L^2 m}{h^2}\epsilon$

6. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E=0$, ϵ and 2ϵ having degeneracies $g(E)=1, 2$ and 4 respectively. The mean energy of the system is:
(a) $\epsilon\left(\frac{4\exp\left(-\epsilon/k_B T\right)+8\exp\left(-2\epsilon/k_B T\right)}{1+2\exp\left(-\epsilon/k_B T\right)+4\exp\left(-2\epsilon/k_B T\right)}\right)$ (b) $\epsilon\left(\frac{2\exp\left(-\epsilon/k_B T\right)+8\exp\left(-2\epsilon/k_B T\right)}{1+2\exp\left(-\epsilon/k_B T\right)+4\exp\left(-2\epsilon/k_B T\right)}\right)$
(c) $\epsilon\left(\frac{2\exp\left(-\epsilon/k_B T\right)+4\exp\left(-2\epsilon/k_B T\right)}{1+2\exp\left(-\epsilon/k_B T\right)+4\exp\left(-2\epsilon/k_B T\right)}\right)^2$ (d) $\epsilon\left(\frac{\exp\left(-\epsilon/k_B T\right)+2\exp\left(-2\epsilon/k_B T\right)}{1+\exp\left(-\epsilon/k_B T\right)+\exp\left(-2\epsilon/k_B T\right)}\right)$

7. Thermodynamic variables of a system can be volume V, pressure P, temperature T, number of particles N, internal energy E and chemical potential μ, etc. For a system to be specified by microcanonical (MC), Canonical (CE) and Grand Canonial (GC) ensembles, the parameters required for the respective ensembles are
(a) MC : (N, V, T); CE (E, V, N); GC : (V, T, μ)
(b) MC : (E, V, N); CE : (N, V, T); GC : (V, T, μ)
(c) MC : (V, T, μ); CE : (N, V, T); GC : (E, V, N)
(d) MC : (E, V, N); CE : (V, T, μ); GC : (N, V, T)

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8. The partition function of a single oscillator with energy levels $\left(n+\frac{1}{2}\right)\hbar\omega$ is given by

(a) $Z=\frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$ (b) $Z=\frac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$ (c) $Z=\frac{1}{1-e^{-\beta\hbar\omega}}$ (d) $Z=\frac{1}{1+e^{-\beta\hbar\omega}}$

9. The average number of energy quanta of the oscillators is given by

(a) $\langle n\rangle=\frac{1}{e^{-\beta\hbar\omega}-1}$ (b) $\langle n\rangle=\frac{e^{-\beta\hbar\omega}}{e^{-\beta\hbar\omega}-1}$ (c) $\langle n\rangle=\frac{1}{e^{\beta\hbar\omega}+1}$ (d) $\langle n\rangle=\frac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

10. A system has energy level $E_0, 2E_0, 3E_0, \ldots\ldots$, where the excited states are triply degenerate. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is:

(a) 2 (b) 3 (c) 4 (d) 5

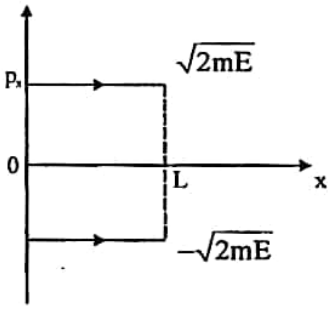
11. It is necessary to apply quantum statistics to a system of particles if
(a) There is substaintial overlap between the wavefunctions of the particles.
(b) The mean free path of the particles is comparable to the inter-particle separation.
(c) The particles have identical mass and charge.
(d) The particles are interacting.

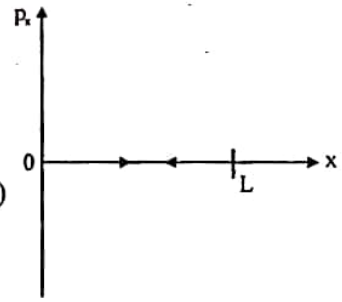
Consider a radiation cavity of volume V at temperature T.

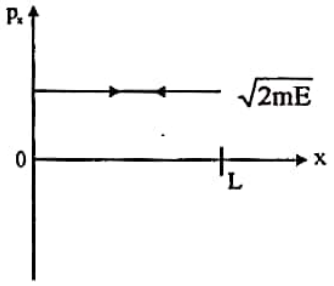
12. A monatomic crystalline solid comprises of N atoms, out of which n atoms are in interstitial positions. If the available interstitial sites are N, then number of possible microstates is:

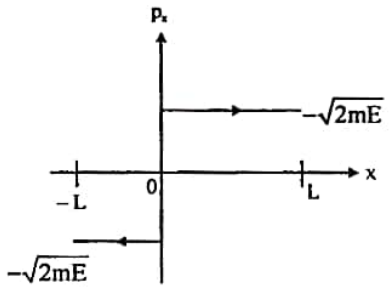
(a) $\frac{(N'+n)!}{n!N!}$ (b) $\frac{(N+n)!}{n!(N'-n)!}\frac{(N'+n)!}{n!(N'-n)!}$ (c) $\frac{N!}{n!(N'-n)!}$ (d) $\frac{N!}{n!(N-n)!}\frac{N'!}{n!(N'-n)!}$

13. The phase diagram of a free particle of mass m and kinetic energy E, moving in one-dimensional box with perfectly elastic walls at x = 0 and x = L, is given by

(a)


(b)


(c)


(d)


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14. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at a pressure P. The temperature of A and B are T_1 and T_2, respectively. The two vessels are brought into thermal contct. At equlibrium, the change in entropy is:

(a) $\frac{3}{2}Nk_B \ln\left[\frac{T_1^2+T_2^2}{4T_1T_2}\right]$ (b) $Nk_B \ln\left(\frac{T_2}{T_1}\right)$

(c) $\frac{3}{2}Nk_B \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (d) $2Nk_B$

15. The number of states for a system of N identical free particles in a three dimensional space having total energy between E and $E+\delta E(\delta E << E)$, is proportional to

(a) $\left(E^{\frac{3N}{2}-1}\right)\delta E$ (b) $E^{\frac{N}{2}}\delta E$ (c) $NE^{\frac{1}{2}}\delta E$ (d) $N^N\delta E$

16. The total number of accessible states of N non intracting particles of spin 1/2 is
(a) 2^N (b) N^2 (c) $2N^{N/2}$ (d) N

17. The dimension of phase space of ten rigid diatomic molecules is:
(a) 5 (b) 10 (c) 50 (d) 100

18. Which of the following relations between the particle number density n and temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?
(a) $n/T^{1/2} << 1$ (b) $n/T^{3/2} << 1$
(c) $n/T^{3/2} >> 1$ (d) $n/T^{1/2}$ and $n/T^{3/2}$ can have any value.

19. In a classical micro-canonical ensemble for a system of N non-interacting particles, the fundamental volume in phase space which is regarded as "equivalent to one micro-state" is
(a) h^{3N} (b) h^{2N} (c) h^N (d) h
where h is the Planck's constant.

20. Which of the following statements is true?
(a) In a micro-canonical ensemble the total number of particles N and the energy E are constants while in a canonical ensemble N and temperature T are constants.
(b) In a micro-canonical ensemble the total number of particles N is a constant but the energy E is variable while in a canonical ensemble N and T are constants.
(c) In a macro-canonical ensemble N and E are constants while in a canonical ensemble N and T both vary.
(d) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N is a constant but T varies.

21. Two particles are said to be distinguishable when
(a) the average distance between them is large compared to their de Broglie wavelength.
(b) the average distance between them is small compared to their de Broglie wavelengths.
(c) they have overlapping wavepackets.
(d) their total wave function is symmetric under particle exchange.

22. In a canonical ensemble
(a) the energy and the temperature are constants.
(b) the entropy and the energy are constans.
(c) the temperature and the density are constants.
(d) the density and the entropy are constants.

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23. A cavity contains blackbody radiation in equilibrium at temperature T. The specific heat per unit volume of the photon gas in the cavity is of the form $C_V = \gamma T^3$ where γ is a constant. The cavity is expanded to twice its original volume and then allowed to equilibrate at the same temperature T. The new internal energy per unit volume is:

(a) $4\gamma T^4$ (b) $2\gamma T^4$ (c) $\frac{\gamma T^4}{4}$ (d) γT^4

24. The distribution function $f(\varepsilon)$ for a photon gas is given by

(a) $\exp(-\varepsilon/kT)$ (b) $\frac{1}{\exp(\varepsilon/kT)+1}$

(c) $\frac{1}{\exp(\varepsilon/kT)-1}$ (d) $\frac{1}{\exp(-\varepsilon/kT)-1}$

25. Consider the electron gas in a simple metal at room temperature
(a) This is a degenerate Fermi-Dirac gas.
(b) The gas displays paramagnetism owing to the spins of the electrons.
(c) All the electrons are in the same single-particle energy level.
(d) The electrons have wave functions localized about the lattice positions of the positive ions.

26. A cavity of volume V contains blackbody radiation at an absolute temperature T.
(a) The internal energy of the radiation is proportional to T^4.
(b) The specific heat of radiation is proportional to T^3.
(c) The pressure of the radiation is proportional to T^4
(d) Although the radiation is in thermal equailibrium, the number of photons in the cavity is not constant in time.

27. Bose Einstein condensation temperature T_B refers to the temperature below which
(a) an assembly of Bose gas condenses to the liquid state.
(b) there is an appreciable occupation of the ground state in an electron system.
(c) there is a significantly large occupancy of the ground state in a system of bosons
(d) the bosons essentially behave like fermious.

28. The energy of a single particle in an infinite potential box is $E_0 n^2$ where $E_0 = \hbar^2 \pi / 2ma^2, n = 1,2,....$ If three non-interacting bosons are in such a potential box, the ground state energy of the system will be
(a) $3E_0$ (b) $4E_0$ (c) $6E_0$ (d) $9E_0$

29. The Fermi energy of a free electron gas depends on the electron density ρ as,
(a) $\rho^{1/3}$ (b) $\rho^{2/3}$ (c) $\rho^{-1/3}$ (d) $\rho^{-2/3}$

30. For an energy state E of a photon gas, the density of states is proportional to
(a) $\sqrt{3}$ (b) E (c) $E^{1/2}$ (d) E^2

31. Consider the Fermi-Dirac distribution function f(E) at room temperature (300 K) where E refers to energy. If E_F is the Fermi energy, which of the following is true?
(a) f(E) is a step function (b) $f(E_F)$ has a value of 1/2
(c) States with $E < E_F$ are filled completely
(d) f(E) is large and tends to infinity as E decreases much below E_F.

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32. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place?

(a) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{2/3}$ (b) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

(c) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{V\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$ (d) $T < \frac{h^2}{2\pi m k_B}\left\{\frac{N}{N\varsigma\left(\frac{3}{2}\right)}\right\}^{1/2}$

where m is the mass of each particle of the system, k_B is the Boltzmann constant, h is the Planck's constant and ς is the well known Zeta function.

33. Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 cm^3 to 640 cm^3, the temperature of the cavity changes to
(a) 800 K (b) 700 K (c) 600 K (d) 500 K

34. The specific heat of an ideal Fermi gas in 3-dimension at very low temperatures (T) varies as
(a) T (b) $T^{3/2}$ (c) T^2 (d) T^3.

A gas of N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N \ln\left[(V - bN)(k_B T)^{3/2}\right]$ where b is a constant with appropriate dimensions.

35. If P is the pressure of the gas, the equation of state is given by
(a) $P(V - bN) = Nk_B T$ (b) $P(V - bN) = k_B T$
(c) $P(V - b) = Nk_B T$ (d) $P(V - b) = Nk_B T$

36. The internal energy of the gas is given by
(a) $U = (1/2)k_B T$ (b) $U = (1/2)k_B T$ (c) $U = (3/2)Nk_B T$ (d) $U = 2Nk_B T$

37. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?
(a) Its energy and the number of photons both increase 8 times.
(b) Its energy increases 8 times and the number of photons increase 16 times.
(c) Its energy increases 16 times and the number of photons increases 8 times.
(d) Its energy and the number of photons both increases 16 times.

38. The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3}aVT^4$ where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is
(a) 0 (b) $\frac{4}{3}aVT^3$ (c) $\frac{1}{3}aVT^3$ (d) aVT^{-4}

39. A one dimensional random walker takes steps to left or right with equal probability. The probability that the random starting from origin is back to origin after N even number of steps is:
(a) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$ (c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

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40. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{kT}{h\omega}$, then the free energy of N such independent oscillators is:

(a) $\frac{3}{2}NkT$ (b) $kT\ln\frac{\hbar\omega}{kT}$ (c) $NkT\ln\frac{\hbar\omega}{kT}$ (d) $NkT\ln\frac{\hbar\omega}{2kT}$

41. The partition function of two Bose particles each of which can occupy any of the two energy levels 0 and ε is:

(a) $1+e^{-2\varepsilon/kT}+2e^{-\varepsilon/kT}$ (b) $1+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (c) $2+e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$ (d) $e^{-2\varepsilon/kT}+e^{-\varepsilon/kT}$

42. A system of non-interacting Fermi particles with Fermi energy E_F has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature T = 0 is:

(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$ (c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$

43. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is:

(a) $P=\frac{3}{2}\frac{U}{V}$ (b) $P=\frac{2}{3}\frac{U}{V}$ (c) $P=\frac{3}{5}\frac{U}{V}$ (d) $P=\frac{1}{2}\frac{U}{V}$

An ensemble of N three-level systems with energies $E=-\epsilon_0, 0, +\epsilon_0$ is in thermal equilibrium at temperature T. Let $\beta=(k_\beta T)^{-1}$.

44. If $\beta\varepsilon_0 = 2$, the probability of finding the system in the level $\varepsilon = 0$ is:

(a) $(\cos h\,2)/2$ (b) $(\cos h\,2)^{-1}$ (c) $(2\cos h\,2)^{-1}$ (d) $(1+2\cos h\,2)^{-1}$

45. The free energy of the system at high temperature (i.e., $x \equiv \beta\varepsilon_0 << 1$) is approximately

(a) $-NK_B Tx^2$ (b) $-NK_B T\left[\ln 2 + x^2/2\right]$

(c) $-NK_B T\left[\ln 3 + x^2/3\right]$ (d) $-NK_B T\ln 3$

46. A system of N localized, non-interacting spin $\frac{1}{2}$ ions of magnetic moment μ each is kept in an external magnetic field H. If the system is in equilibrium at temperature T, then Helmholtz free energy of the system is:

(a) $Nk_B T\ln\left(\cos h\frac{\mu H}{k_B T}\right)$ (b) $-Nk_B T\ln\left(2\cos h\frac{\mu H}{k_B T}\right)$

(c) $Nk_B T\ln\left(2\cos h\frac{\mu H}{k_B T}\right)$ (d) $-Nk_B T\ln\left(2\sin h\frac{\mu H}{k_B T}\right)$

47. The free energy for a photon gas is given by $F=-\left(\frac{a}{3}\right)VT^4$, where *a* is a constant. The entropy S and the pressure P of the photon gas are

(a) $S=\frac{4}{3}aVT^3, P=\frac{a}{3}T^4$ (b) $S=\frac{1}{3}aVT^4, P=\frac{4a}{3}T^3$

(c) $S=\frac{4}{3}aVT^4, P=\frac{a}{3}T^3$ (d) $S=\frac{1}{3}aVT^3, P=\frac{4a}{3}T^4$

48. The density of states at energy E of the quantized radiation (photons) is:

(a) $\frac{8\pi V}{h^3C^3}E^2$ (b) $\frac{8\pi V}{h^3C^3}E^{1/2}$ (c) $\frac{8\pi V}{h^3C^3}E$ (d) $\frac{8\pi V}{h^3C^3}E^{1/2}$

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49. The average number of photons in equilibrium inside the cavity is proportional to
(a) T (b) T^2 (c) T^3 (d) T^4.

50. A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = \hbar\omega$ is:

(a) $\exp\left(\frac{\hbar\omega}{k_B T}\right)+1$ (b) $\exp\left(\frac{\hbar\omega}{k_B T}\right)-1$ (c) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right)+1\right)^{-1}$ (d) $\left(\exp\left(\frac{\hbar\omega}{k_B T}\right)-1\right)^{-1}$

51. Two identical particles have to be distributed among three energy levels. let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given enegy state. The subscripts B, E and C correspond to whether the particles are bosons, fermions and classical particles, respectively. The $r_B:r_F:r_C$ is equal to

(a) $\frac{1}{2}:0:1$ (b) $1:\frac{1}{2}:1$ (c) $1:\frac{1}{2}:\frac{1}{2}$ (d) $1:0:\frac{1}{2}$

52. The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electrons N is given by

(a) $\frac{1}{3}NE_F$ (b) $\frac{1}{2}NE_F$ (c) $\frac{2}{3}NE_F$ (d) $\frac{3}{5}NE_F$

53. Across a first order phase transition, the free energy is
(a) proportional to the temperature
(b) a discontinuous function of the temperature
(c) a continuous function of the temperature but its derivative is discontinuous
(d) such that the first derivative with respect to temperature is continuous

54. Two gases separated by an impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same
(a) pressure and temperature (b) volume and temperature
(c) pressure and volume (d) volume and energy

55. The entropy function of a system is given by $S(E) = aE(E_0 - E)$ where a and E_0 are positive constants. The temperature of the system is
(a) negative for some energies (b) increases monotonically with energy
(c) decreases monotonically with energy (d) zero

56. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant)

(a) Zero (b) Nk (c) $\frac{1}{2}Nk$ (d) $Nk\ ln(2)$

57. The number of distinct ways of placing four indistinguishable balls into five distinguishable boxes is _____

58. For a gas under isothermal conditions, its pressure P varies with volume V as $P \propto V^{-5/3}$. The bulk modulus B is proportional to

(a) $V^{-1/2}$ (b) $V^{-2/3}$ (c) $V^{-3/5}$ (d) $V^{-5/3}$

59. At a given temperature, T, the average energy per particle of a non-interacting gas of two-dimensional classical harmoic oscillator is _____ $k_B T$.

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Answer Key (STATISTICAL-I)

1. (b)	2. (a)	3. (d)	4. (c)	5. (a)	6. (b)	7. (b)
8. (b)	9. (a)	10. (b)	11. (a)	12. (d)	13. (a)	14. (c)
15. (a)	16. (a)	17. (b)	18. (c)	19. (a)	20. (a)	21. (a)
22. (c)	23. (d)	24. (c)	25. (a)	26. (d)	27. (c)	28. (a)
29. (b)	30. (d)	31. (b)	32. (a)	33. (d)	34. (a)	35. (a)
36. (c)	37. (c)	38. (a)	39. (a)	40. (c)	41. (b)	42. (d)
43. (b)	44. (d)	45. (d)	46. (b)	47. (a)	48. (a)	49. (c)
50. (d)	51. (d)	52. (b)	53. (b)	54. (a)	55. (a)	56. (d)
57. (70)	58. (d)	59. (1.99 to 2.01)				

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CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED
Institute for NET-JRF/GATE & IIT-JAM

PHYSICAL SCIENCES (JRF-NET/GATE)

Assignment-II: STATISTICAL

1. A system in thermal equilibrium consists of subsystems A and B that interact only weakly with each other. If Z_A and Z_B are the canonical partition functions of A and B respectively, the partition function of the total system is given, to a good approximation, by

   (a) $Z_A + Z_B$ (b) Z_A / Z_B (c) $Z_A - Z_B$ (d) $Z_A Z_B$

2. The standard deviation of the energy of a system in the canonical ensemble is equal to

   (a) $\sqrt{C_v}$ (b) $k_B T^2 C_v$ (c) $\sqrt{k_B T^2 C_v}$ (d) $\sqrt{k_B T C_v}$

3. Let N be the total number of distinguishable particles in a system, out of which the number of particles with energies $E_1, E_2, \ldots\ldots E_n$ is $N_1, N_2 \ldots\ldots N_n$ respectively, $(N_1 + N_2 + \ldots.. N_n = N)$
   The number of ways in which these particles can be distributed is

   (a) $\dfrac{N!}{N_1! N_2! - - - N_n!}$ (b) $\dfrac{N_1! N_2! - - - N_n!}{N!}$

   (c) $\dfrac{N!}{\sqrt{N_1!} + \sqrt{N_2!} - - - \sqrt{N_n!}}$ (d) $\dfrac{\sqrt{N!}}{N_1! N_2! - - - N_n!}$

4. The entropy of a system in the microcanonical ensemble formalism
   (a) Is independent of the number of microslates.
   (b) Is directly proportional to the number of microstates.
   (c) Is inversely proportional to the number of microstates.
   (d) Increases logarithmicaly with increase in number of microstates.

5. A degenerate electron gas is characterised by the following property:
   (a) Each energy level has a very large occupancy.
   (b) Each energy level has only a single occupancy.
   (c) High density and low temperature.
   (d) Low density and high temperature.

6. Z_1, Z_2 and Z_3 represent translational, vibrational and rotational partition functions of a diatomic molecule. The partition function of the molecule Z is given by

   (a) $Z = Z_1 Z_2 Z_3$ (b) $Z = Z_1 + Z_2 + Z_3$

   (c) $\ln Z = \ln(Z_1 + Z_2 + Z_3)$ (d) $e^Z = e^{Z_1} + e^{Z_2} + e^{Z_3}$

7. The principle of equal a priori probability asserts than
   (a) All states of a system are equally probable.
   (b) All microstates of a system compatible with a macrostate of energy between E and $E + \Delta E$ are equally probable.
   (c) All microstates of a system which are incompatible with a macrostate energy E are equally probable.
   (d) A-priori probability of all microstates has a normal distribution.

8. In a statistical description of a system in the canonical ensemble, the relative uncertainty in the energy of the system, $\Delta E/E$, varies with the number of particles N in the systems as

   (a) N^{-1} (b) $N^{-1/2}$ (c) N^0 (d) $N^{1/2}$

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9. In a canonical ensemble description of a system, which of the following quantities remain fixed?
(a) Energy of the system (b) Square of the energy of the system
(c) Volume of the system (d) Number of particles in the system.

10. E_1 and E_2 is non degenerate energy then probability of E_2.
(a) $\frac{\exp(\beta E_1)}{\exp(\beta E_1)+\exp(\beta E_2)}$ (b) $\frac{\exp(\beta E_1)+\exp(\beta E_2)}{\exp(-\beta E_1)+\exp(-\beta E_2)}$
(c) $\frac{\exp(-\beta E_1)}{\exp(-\beta E_2)+\exp(-\beta E_1)}$ (d) $\frac{\exp(-\beta E_2)}{\exp(-\beta E_1)+\exp(-\beta E_2)}$

11. A system has two energy levels 0 and e > 0. The energy levels are populated according to the Boltzmann's distribution law. In the limit $T \to \infty$ the entropy per particle is:
(a) $+\infty$ (b) $k_B \ln 2$ (c) $2k_B$ (d) $k_B \ln e^2$

12. An ensemble of identical harmonic oscillators with aHamiltonian, $H = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$ have energies between E and $E + \Delta E$. If the oscillators are initially, uniformly distributed in the phase space then which of the following is false?
(a) More particles are at the maximum amplitude than at the origin
(b) The mean displacement is zero.
(c) Mean kinetic energy is equal to the mean potential energy.
(d) More particles are at the origin than at maximum amplitude.

13. N molecules of an ideal gas are in a container of volume V_0. If a molecule has the same probability of being at any point in the container, the probability of M of them being in a smaller volume V in the container is:
(a) M/N (b) $\frac{V^M (V_0 - V)^{N-M}}{V_0^N}$
(c) $\frac{N!}{(N-M)!M!}\frac{V^M (V_0 - V)^{N-M}}{V_0^N}$ (d) $\frac{N!}{(N-M)!M!}\frac{V_0^M (V_0 - V)^{N-M}}{V^N}$

14. A non-interacting system has two energy levels, 0 and ε. The lower level is doubly degenerate while that of energy ε is non-degenerate. If the system is in thermal equilibrium at temperature T, the single - particle partitin function is:
(a) $Z = 2e^{-\varepsilon/k_B T}$ (b) $Z = 1 + 2e^{-\varepsilon/k_B T}$ (c) $Z = 2 + e^{-\varepsilon/k_B T}$ (d) $Z = 2\left(1 + e^{-\varepsilon/k_B T}\right)$

15. A system of weakly interacting two-dimensional harmonic oscillators is in thermal conact with a heat bath of absolute temperature T. The average kinetic energy of an oscillator is:
(a) $k_B T$ (b) $2k_B T$ (c) $3k_B T/2$ (d) $k_B T/2$

16. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is:
(a) $2k_B \ln N$ (b) $3k_B \ln N$ (c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

17. A system has two energy levels with energies ε and 2ε. The lower level is 4-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is:
(a) $\frac{1}{1+e^{-\varepsilon/k_B T}}$ (b) $\frac{1}{1+2e^{\varepsilon/k_B T}}$ (c) $\frac{1}{2e^{\varepsilon/k_B T}+4e^{2\varepsilon/k_B T}}$ (d) $\frac{1}{2e^{\varepsilon/k_B T}-4e^{2\varepsilon/k_B T}}$

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18. Consider a system of N non-interacting spins, each of which has classifical magnetic moment of magnitude μ. The Hamiltonian of this system in an external magnetic field $\bar{H}$ is $\mathcal{H} = -\sum_{i=1}^{N} \bar{\mu}_i . \bar{H}$, where $\bar{\mu}_i$ is the magnetic moment of the i^{th} spin. The magnetization per spin at temperature T is:

(a) $\frac{\mu^2 H}{k_B T}$ (b) $\mu\left[\coth\left(\frac{\mu H}{k_B T}\right) - \frac{k_B T}{\mu H}\right]$

(c) $\mu \sinh\left(\frac{\mu H}{k_B T}\right)$ (d) $\mu \tanh\left(\frac{\mu H}{k_B T}\right)$

19. A gas of N non-interacting particles is in thermal equilibrium at temperature T. Each particle can be in any of the possible non-degenerate states of energy 0, 2ε and 4ε. The average energy per particle of the gas, when $\beta\varepsilon \ll 1$, is:

(a) 2ε (b) 3ε (c) $2\varepsilon/3$ (d) ε

20. A one-dimensional chain consists of a set of N rods each of length a. When stretched by a load, each rod can align either parallel or perpendicular to the length of the chain. The energy of a rod is $-\varepsilon$ when aligned parallel to the length of the chain and is $+\varepsilon$ when perpendicular to it. When the chain is in thermal equilibrium at temperature T, its average length is:

(a) Na/2 (b) Na (c) $Na/\left(1+e^{-2\varepsilon/k_B T}\right)$ (d) $Na/\left(1+e^{-2\varepsilon/k_B T}\right)$

21. Consider a system with two allowed microstates, one at energy zero and the other at energy $k_B T$ The system is in equilibrium at temperature T. What is the probability of finding the system in its ground state?

(a) 1 (b) $\frac{1}{e}$ (c) $\frac{1}{1+e}$ (d) $\frac{e}{1+e}$

22. Let $\sigma_v^2 \equiv \left\langle U - \langle U \rangle^2 \right\rangle$ and $\sigma_v^2 = \left\langle P - \langle P \rangle^2 \right\rangle$ be the variances of the internal energy U and the pressure P about their mean values $\langle U \rangle$ and $\langle p \rangle$ in a system of N particles at constant temperature. Then

(a) $\sigma_U^2 \propto N$ and $\sigma_p^2 \propto N$ (b) $\sigma_U^2 \propto \frac{1}{N}$ and $\sigma_p^2 \propto \frac{1}{N}$

(c) $\sigma_U^2 \propto \frac{1}{N}$ and $\sigma_p^2 \propto N$ (d) $\sigma_U^2 \propto N$ and $\sigma_p^2 \propto \frac{1}{N}$

23. In the canonical ensemble description, a system
(a) can only exchange energy with its environment
(b) is isolated from its environment
(c) can exchange energy as well as particles with its environment
(d) can only exchange particles with its environment

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24. A closed system can be in any one of two nondegenerate microstates with energy E_1 and E_2. The system is in thermal equilibrium with a heat bath at temperature T. The probability that the system will be observed in the state with energy E_2 is given by

(a) $\exp\left(-\frac{E_2}{k_BT}\right)$ (b) $\exp\left(-\frac{E_1-E_2}{k_BT}\right)$

(c) $\frac{\exp\left(-\frac{E_2}{k_BT}\right)}{\exp\left(-\frac{E_1}{k_BT}\right)+\exp\left(-\frac{E_2}{k_BT}\right)}$ (d) $\frac{\exp\left(-\frac{E_1-E_2}{k_BT}\right)}{\exp\left(-\frac{E_1}{k_BT}\right)+\exp\left(-\frac{E_2}{k_BT}\right)}$

25. Consider a system whose three energy levels are given by 0, ε and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta=\frac{1}{k_BT}$ is given by

(a) $1+2e^{-\beta\epsilon}$ (b) $2e^{-\beta\epsilon}+e^{-2\beta\epsilon}$ (c) $\left(1+e^{-\beta\epsilon}\right)^2$ (d) $1+e^{-\beta\epsilon}+e^{-2\beta\epsilon}$

26. Consider a system of particles in three dimensions with momentum $\vec{p}$ and energy $E=c.\vec{P}|$ .c being a constant. The system is maintained at inverse temperature β. volume V and chemical μ. What is the grand particle partition function of the system?

(a) $\exp\left[e^{\beta\mu}8\pi V/(\beta ch)^3\right]$ (b) $e^{\beta\mu}6\pi V/(\beta ch)^2$

(c) $\exp\left[e^{\beta\mu}6\pi V/(\beta ch)^3\right]$ (d) $e^{\beta\mu}8\pi V/(\beta ch)^2$

27. A collection of N two- level systems with energies 0 and E > 0 is in thermal equilibrium at temperature T. For $T\to\infty$, the specific. heat approaches

(a) 0 (b) Nk_B (c) $3Nk_{B/2}$ (d) ∞

28. Consider a system maintained at temperature T, With two available energy states E_1 and E_2 each with degeneracies g_1 and g_2. If p_1 and p_2 are probabilities of occupancy of the energy states, what is the entropy of the system?

(a) $S=-\kappa_B\left[p_1\ln(p_1/g_1)+p_2\ln(p_2/g_2)\right]$ (b) $S=-\kappa_B\left[p_1\ln(p_1g_1)+p_2\ln(p_2g_2)\right]$

(c) $S=-\kappa_B\left[p_1\ln\left(p_1^{g_1}\right)+p_2\ln\left(p_2^{g_2}\right)\right]$ (d) $S=-\kappa_B\left[(1/p_1)\ln(p_1/g_1)+(1/p_2)\ln(p_2/g_2)\right]$

29. A system has two normal modes of vibration, with frequencies ω_1 and $\omega_2=2\omega_1$. What is the probability that at temperature T, the system has an energy less than $4\hbar\omega_1$? [In the following $x=e^{-\beta\hbar\omega}$ and Z is the partition function]

(a) $x^{3/2}\left(x+2x^2\right)/Z$ (b) $x^{3/2}\left(1+x+x^2\right)/Z$

(c) $x^{3/2}\left(1+2x^2\right)/Z$ (d) $x^{3/2}\left(1+x+2x^2\right)/Z$

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30. The entropy of a system, S, is related to the accessible phase space volume Γ by $S = k_B \ell n\, \Gamma(E, N, V)$ where E, N and V are the energy, number of particles and volume respectively. From this one can conclude that Γ
(a) does not change during evolution to equilibrium
(b) Oscillates during evolution to equilibrium
(c) Is a maximum in equilibrium
(d) Is a minimum in equilibrium

31. Consider a one-dimensional Ising model with N spins, at very low temperatures when almost all the spins are aligned parallel to each other. There will be a few spin flips with each flip costing an energy 2J. In a configuration with r spin flips, the energy of the system is $E = -NJ + 2rJ$ and the number of configuration is ${}^{N}C_{r}$; r varies from 0 to N. The partition function is

(a) $\left(\frac{J}{k_B T}\right)^N$ (b) $e^{-NJ/k_B T}$ (c) $\left(\sinh\frac{J}{k_B T}\right)^N$ (d) $\left(\cosh\frac{J}{k_B T}\right)^N$

32. Non-interacting bosons undergo Bose-Einstein Condensation (BEC) when trapped in a three-dimensional isotropic simple harmonic potential. For BEC to occur, the chemical potential must be equal to
(a) $\hbar\omega/2$ (b) $\hbar\omega$ (c) $3\hbar\omega/2$ (d) 0

33. Bose condensation occurs in liquid He^4 kept at ambient pressure at 2.17 K. At which temperature will Bose condensation occur in He^4 in gaseous state, the density of which is 1000 times smaller than that of liquid He^4? (Assume that it is a perfect Bose gas)
(a) 2.17 mK (b) 21.7 mK (c) 21.7 μK (d) 2.17 μK

34. A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that N/2spins are aligned parallel and the remaining N/2 spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is,
(a) $2Nk_B \ell n\, 2$ (b) $\frac{N}{2}k_B \ell n\, 2$ (c) $\frac{3N}{2}k_B \ell n\, 2$ (d) $Nk_B \ell n\, 2$

35. The total energy, E of an ideal non-relativistic Fermi gas in three dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$ where N is the number of particles and V is the volume of the gas. Identify the CORRECT equation of state (P being the pressure),
(a) $PV = \frac{1}{3}E$. (b) $PV = \frac{2}{3}E$ (c) $PV = E$ (d) $PV = \frac{5}{3}E$

36. If the volume of a three- dimensional free Fermi gas is decreased by a factor of 8 at zero temperature. its internal energy changes by a factor of
(a) 8 (b) 2 (c) 4 (d) $\frac{1}{8}$

37. An ideal Bose gas undergoes Bose-Einstein condensation at temperature T_0. At any finite temperature below T_0,
(a) the chemical potential is equal to zero
(b) only the lowest energy level is occupied
(c) the internal energy is equal to zero
(d) the pressure is equal to zero.

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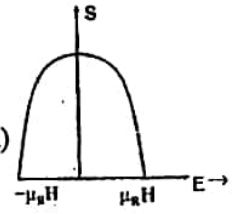
38. Two non-interacting bosons are distributed in three degenerate levels of energy zero. The canonical partition function of this system is
(a) 4.5 (b) 6 (c) 9 (d) 8

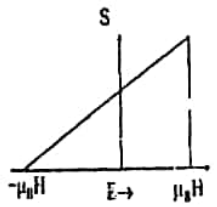
39. If the volume of a three-dimensional system of non-interacting fermions is decreased by a factor of 27 at zero temperature, its, internal energy change by a factor of
(a) 9 (b) 3 (c) 1/27 (d) 1/9

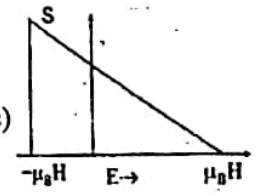
40. Consider a system of a large number N of very weakly interacting spin-1/2 particles, each with magnetic moment μ. A magnetic field H is applied in the z-direction. The ratio between the numbers of up-and down-spin particles is given by
(a) $(1+\beta\mu H):1$ (b) $(1-\beta\mu H):1$ (c) $1:e^{\beta\mu H}$ (d) $e^{\beta\mu H}:1$

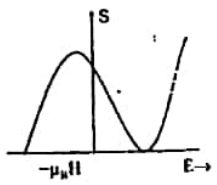
41. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temprature will
(a) increase by a factor of 4 (b) decrease by a factor of 4
(c) increase by a factor of 8 (d) decrease by a factor of 8

42. A system of N noninteracting spin $-\frac{1}{2}$ particles is placed in an external magnetic field H. The behaviour of the entropy of the system as a function of energy is given by

(a) 

(b) 

(c) 

(d) 

43. The energy density of black-body radiation at absolute temperature T in d dimensions proportional to
(a) $T^{(d^2-1)/2}$ (b) T^{d+1} (c) $T^{(d+5)/2}$ (d) T^{2d-2}

44. Consider an ideal Bose gas in three dimensions with the energy-momentum relation $\varepsilon \propto p^s$ with $s>0$. The range of s for which this system may undergo a Bose-Einstein condensation at a non-zero temperature is:
(a) $1<s<3$ (b) $0<s<2$ (c) $0<s<3$ (d) $0<s<\infty$

45. The chemical potential p is:
(a) Positive in Fermi-Dirac statistics (FD) and positive in Bose-Einstein statistics (BE) at room temperature (RT)
(b) Positive in FD and negative in BE at RT
(c) Negative in Maxwell Boltzmann statistics and negative in FD at RT.
(d) Always positive in all the three statistics.

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46. Considering fermionic particles A and B with chemical potentials μ and $-\mu$ respectively moving in a box at temperature T. The number density of particle A is:
(a) greater than that of B (b) less than that of B
(c) equal to that of B (d) zero.

47. Fermi energy E_F of an electron gas of N electrons confined in a volume V is proportional to:
(a) $(N/V)^{2/3}$ (b) $(N/V)^{1/3}$ (c) $(V/N)^{1/3}$ (d) $(V/N)^{2/3}$

48. A gas of N spin 1/2 particles and magnetic moment $\bar{\mu}$ each, is kept in uniform magnetic induction $\bar{B}$ at temperature T. Then, average energy of the gas is:
(a) $N\mu B\, e^{-\beta\mu B}$ (b) $-N\mu B\, e^{\beta\mu B}$
(c) $N\mu B\left(e^{\beta\mu B}+e^{-\beta\mu B}\right)$ (d) $\dfrac{-N\mu B\left(e^{-\beta\mu B}-e^{\beta\mu B}\right)}{\left(e^{-\beta\mu B}+e^{\beta\mu B}\right)}$

Where $\beta = 1/k_B T$; k_B is Boltzmann constant.

49. It is observed that the highest occupied energy level in the ground state of the system of non-interacting identical particles enclosed in a box increases with the number of particles. From this observation, we can conclude that the particles have
(a) Zero spin (b) Odd integer spin (c) Even integer spin (d) Half integer spin

50. The difference between Maxwell-Boltzmann (MB), and Bose-Einstein (BE) distribution is that
(a) Both are identical except that one is for classical particles and the other for quantum particles.
(b) Density of occupied states at lower energies is more for BE than for MB.
(c) Density of occupied states at lower energies is less for BE than for MB.
(d) MB is applicable to particles with zero spin and BE to those of finite spin.

51. A system of particles can undergo a Bose Einstein condensation if all particles
(a) Have zero rest mass and zero spin (b) Have half-integer spins
(c) Have mass and spin zero (d) Are free electrons.

52. A particle of spin $\frac{1}{2}$ and magnetic moment μ is in a magnetic field B. Its partition function is:
(a) $2\tanh \mu B/kT$ (b) $2\cosh\dfrac{\mu B}{kT}$ (c) $2\sin h\dfrac{\mu B}{kT}$ (d) $e^{\frac{\mu B}{kT}}$

53. For blackbody radiation.
(a) The temperature of the photon gas is lower than the temperature of its container.
(b) The temperature of the photon gas is higher than the temperature of the container.
(c) The temperature of the electromagnetic radiation is a meaningless concept.
(d) The total number of photons in the container is not constant.

54. A system consists of two Bosons each of which can occupy two energy levels 0 and E_0. The partition function for the system is:
(a) $1+e^{-\beta E_0}$ (b) $1+e^{-2\beta E_0}$ (c) $1+e^{-\beta E_0}+e^{-2\beta E_0}$ (d) $e^{-\beta E_0}+e^{-2\beta E_0}$, Where $\beta=\dfrac{1}{kT}$.

55. According to Bose-Einstein Statistics, there exists a 'Bose Condensate' which is a collections of bosons. For this condense, which of the following statements is correct?
(a) As $T\to\infty$, all bosons reside in excited states
(b) For $T < T_c$ almost all bosons reside in the ground state.
(c) As $T\to\infty$, all bosons reside in the ground state.
(d) For $T > T_c$, bosons behave similar to fermions.

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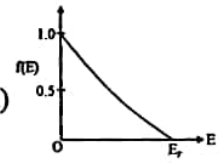
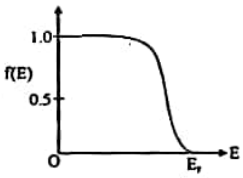
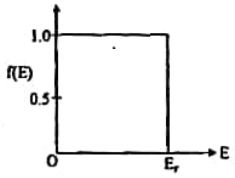
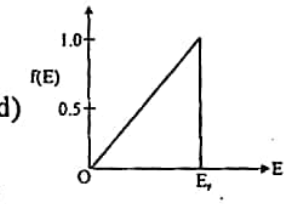
56. According to Fermi-Dirac Statistics, the probability of occupation of a quantum state of energy E is $f(E)=\frac{1}{e^{(E-E_T)/kT}+1}$ where E_F is the Fermi energy. Which of the following grahs is true at T = 0 K.

(a) 

(b) 

(c) 

(d) 

57. Plank's distribution formula black body radiation is given by

$$u_\lambda d_\lambda = \frac{8\pi hc}{-\lambda^5}\frac{d\lambda}{e^{hc/\lambda kT}-1}$$

For long wave lengths and high temperatures, u_λ, d_λ becomes equal to

(a) $\frac{8\pi kT}{\lambda^4}d\lambda$ (b) $\frac{8\pi kT}{\lambda^5}d\lambda$ (c) $\frac{8\pi kT}{\lambda^5}e^{hc/\lambda kT}$ (d) $\frac{8\pi kT}{\lambda^5}\left(e^{-h/\lambda kT}+1\right)$

58. The number of ways in which 5 identical bosons can be distributed in 4 states is:

(a) $\frac{8!}{5!3!}$ (b) $\frac{9!}{5!4!}$ (c) $\frac{9!}{4!4!}$ (d) $\frac{8!}{4!4!}$

59. Blackbody radiation is enclosed inside a spherical cavity of radius r at a temperature T. What would be the temperature of the enclosure if the radius expands to 2r adiabatically?

(a) T/2 (b) T (c) $T/\sqrt{2}$ (d) 2T

60. Two localized non-interacting spin 1/2 magnetic ions of magnetic moment μ are placed in an external magnetic field H, at temperature T. If $k_BT >> \mu H$, then the entropy of the system is, to a good approximation.

(a) $S = k_B \ln 2$ (b) $S = 2k_B \ln 2$ (c) $S = 3k_B \ln 2$ (d) $S = 4k_B \ln 2$

61. If P_1, P_2 and P_3 represent the pressures of ideal B.E.; M.B. and F.D. gases at low temperature, respectively then

(a) $P_1 < P_2$; $P_1 < P_3$ (b) $P_1 < P_3$; $P_1 > P_2$ (c) $P_1 > P_2$; $P_1 > P_3$ (d) $P_1 = P_2 = P_3$

62. For an ideal Fermi gas at absolute Zero, the pressure is:

(a) zero (b) proportional to V
(c) proportional to $V^{-2/3}$ (d) proportional to $V^{-5/3}$.

63. Consider two independently diffusing non- interacting particles in 3-dimensional space, both placed at the origin at time t = 0. These particles have different diffusion constants D_1 and D_2. The quantity $\left\langle\left[\vec{R}_1(t)-\vec{R}_2(t)\right]^2\right\rangle$ where $\vec{R}_1(t)$ and $\vec{R}_2(t)$ are the positions of the particles at time t, behaves as:

(a) $6t(D_1+D_2)$ (b) $6t(D_1-D_2)$ (c) $6t\sqrt{D_1^2+D_2^2}$ (d) $6t\sqrt{D_1D_2}$

64. Consider a random walk of N steps, each of unit length but randomly and independently oriented with respect to each other in a two dimensional space. In the limit $N \to \infty$, the mean square distance of the end point from the starting point is proportional to

(a) N^2 (b) $\sqrt{N}$ (c) N (d) $\frac{1}{N}$

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65. A random walker takes uncorrelated steps along the x-axis with probability $(P > 1/2)$ along the positive x-direction nd with the probability $(1-p)$ along the negative x-direction. After N steps, the mean distance of the walker from the starting position is proportional to :

(a) $N^{1/3}$ (b) $N^{1/2}$ (c) N (d) $N^{4/3}$

Common Data Questions

Common Data for Q. 66 and Q. 67: There are four energy levels E, $2E$, $3E$ and $4E$ (where $E>0$). The canonical partition function of two particles is, if these particles are

66. Two identical fermions

(a) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$ (b) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$

(c) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$ (d) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

67. Two distinguishable particles

(a) $e^{-2\beta E}+e^{-4\beta E}+e^{-6\beta E}+e^{-8\beta E}$ (b) $e^{-3\beta E}+e^{-4\beta E}+2e^{-5\beta E}+e^{-6\beta E}+e^{-7\beta E}$

(c) $(e^{-\beta E}+e^{-2\beta E}+e^{-3\beta E}+e^{-4\beta E})^2$ (d) $e^{-2\beta E}-e^{-4\beta E}+e^{-6\beta E}-e^{-8\beta E}$

68. A vessel has two compartments of volume V_1 and V_2, containing an ideal gas at pressures p_1 and p_2, and temperatures T_1 and T_2 respectively. If the wall separating the compartments is removed, the resulting equilibrium temperature will be

(a) $\dfrac{p_1T_1+p_2T_2}{p_1+p_2}$ (b) $\dfrac{V_1T_1+V_2T_2}{V_1+V_2}$

(c) $\dfrac{p_1V_1+p_2V_2}{(p_1V_1/T_1)+(p_2V_2/T_2)}$ (d) $(T_1T_2)^{1/2}$

69. A system of non-interacting spin-1/2 charged particles are placed in an external magnetic field. At low temperature T, the leading behavior of the excess energy above the ground state energy, depends on T as: (c is a constant)

(a) cT (b) cT^3 (c) $e^{-c/T}$ (d) c(is independent of T)

70. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states D(E) depends on the energy as

(1) $1/\sqrt{E}$ (2) $\sqrt{E}$ (3) E (4) constant

71. Three identical spin $-\dfrac{1}{2}$ fermions are to be distributed in two non-degenerate distinct energy levels. The number of ways this can be done is

(a) 8 (b) 4 (c) 3 (d) 2

72. Consider the melting transition of ice into water at constant pressure. Which of the following thermodynamic quantities does not exhibit a discontinuous change across the phase transition?

(a) internal energy (b) Helmholtz free energy

(c) Gibbs free energy (d) entropy

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73. A system of N classical non-interacting particles, each of mass m, is at a temperature T and is confined by the external potential $V(r)=\frac{1}{2}Ar^2$ (where A is a constant) in three dimensions. The internal energy of the system is

(a) $3Nk_BT$ (b) $\frac{3}{2}Nk_BT$

(c) $N(2mA)^{3/2}k_BT$ (d) $N\sqrt{\frac{A}{m}}\ln\left(\frac{k_BT}{m}\right)$

74. Consider the CO molecule as a system of two point particles which has both translational and rotational degrees of freedom. Using classical statistical mechanics, the molar specific heat C_V of CO gas is given in terms of the Boltzmann constant k_B by

(a) $\frac{5}{2}k_B$ (b) $2k_B$ (c) $\frac{3}{2}k_B$ (d) $\frac{1}{2}k_B$

75. A gas of photons is enclosed in a container of fixed volume at an absolute temperature T. Noting that the photon is massless particle (i.e. its energy and momentum are related by E = pc), the number of photons in the container will vary as

(a) T (b) T^2 (c) T^3 (d) T^4

76. Which of the following statement best explains why the specific heat of electrons in metals is much smaller than that expected in a non-interacting (free) electron gas model?

(a) The mass of electron is much smaller than that of the ions in the crystal.
(b) The Pauli exclusion principle restricts the number of electrons which can absorb thermal energy
(c) Electron spin can take only two different values.
(d) Electrons in a metal cannot be modelled as non-interacting.

77. The root-mean square velocity $\bar{\upsilon}$ of helium atom at T degree K satisfies the relation

(a) $\bar{\upsilon}\propto\sqrt{T}$ (b) $\bar{\upsilon}\propto\frac{1}{\sqrt{T}}$ (c) $\bar{\upsilon}\propto T$ (d) $\bar{\upsilon}\propto\frac{1}{T}$

Common data for Q.78, Q.79 and Q.80.

A system consists of three spin-half particles, the z components of whose spins $S_z(1), S_z(2)$ and $S_z(3)$ can take value +1/2 and −1/2. The total spin of the system is $S_z = S_z(1)+S_z(2)+S_z(3)$

78. The total number of possible micro-states of this system is:
(a) 3 (b) 6 (c) 7 (d) 8

79. The total number of micro-states with $S_z = 1/2$ is:
(a) 3 (b) 5 (c) 6 (d) 7

80. Consider an ensemble of system s where each microstate has equal probability. The ensemble average of S_z is:
(a) −1/2 (b) 0 (c) 1/2 (d) 3/2

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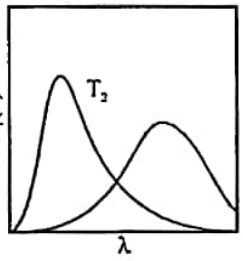
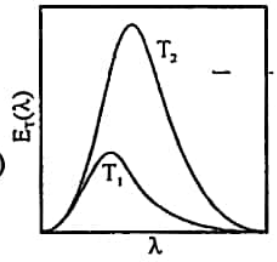
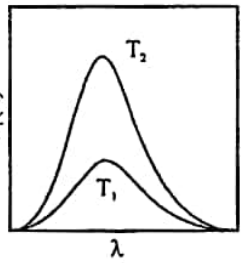
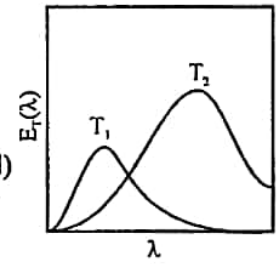
81. Which of the graphs below gives the correct qualitative behavior of the energy density $E_T(\lambda)$ of blackbody radiation of wavelength λ at two temperatures T_1 and T_2 ($T_1 < T_2$)?

(a) 

(b) 

(c) 

(d) 

82. A system can have three energy levels:
$E = 0, \pm\varepsilon$. The level $E = 0$ is doubley degenerate, while the others are non-degenerate. The average energy at inverse temperature β is

(a) $-\varepsilon \tanh(\beta\varepsilon)$

(b) $\dfrac{\varepsilon\left(e^{\beta\varepsilon} - e^{-\beta\varepsilon}\right)}{\left(1 + e^{\beta\varepsilon} + e^{-\beta\varepsilon}\right)}$

(c) Zero

(d) $-\varepsilon \tanh\left(\dfrac{\beta\varepsilon}{2}\right)$

83. For a particular thermodynamics system the entropy S is related to the internal energy U and volume V by

$$S = c\,U^{3/4}\,V^{1/4}$$

where c is a constant. The Gibbs potential $G = U - TS + pV$ for this system is

(a) $\dfrac{3pU}{4T}$ (b) $\dfrac{cU}{3}$ (c) zero (d) $\dfrac{US}{4V}$

84. The pressure of a non-relativistic free Fermi gas in three-dimensions depends, at $T = 0$, on the density of fermions n as

(a) $n^{5/3}$ (b) $n^{1/3}$ (c) $n^{2/3}$ (d) $n^{4/3}$

85. The average local internal magnetic field acting on an Ising spin is $H_{int} = \alpha M$, where M is the magnetization and α is a positive constant. At a temperature T sufficiently close to (and above) the critical temperature T_c, the magnetic susceptibility at zero external field is proportional to (k_B is the Boltzmann constant)

(a) $k_B T - \alpha$ (b) $(k_B T + \alpha)^{-1}$ (c) $(k_B T - \alpha)^{-1}$ (d) $\tanh(k_B T + \alpha)$

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86. The van der Waals equation of state for a gas is given by

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

where, P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by

(a) $\frac{a}{9b}$ (b) $\frac{a}{27b^2}$ (c) $\frac{8a}{27bR}$ (d) $3b$

87. For a system of two bosons, each of which can occupy any of the two energy levels 0 and ε, the mean energy of the system at a temperature T with $\beta=\frac{1}{k_BT}$ is given by

(a) $\frac{\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{1+2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (b) $\frac{1}{2e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

(c) $\frac{2\varepsilon e^{-\beta\varepsilon}+\varepsilon e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$ (d) $\frac{\varepsilon e^{-\beta\varepsilon}+2\varepsilon e^{-2\beta\varepsilon}}{2+e^{-\beta\varepsilon}+e^{-2\beta\varepsilon}}$

88. Consider a system of 3 fermions, which can occupy any of the 4 available energy states with equal probability. The entropy of the system is

(a) $k_B \ln 2$ (b) $2k_B \ln 2$ (c) $2k_B \ln 2$ (d) $3k_B \ln 4$

Answer Key (STATISTICAL-II)

1. (d)	2. (c)	3. (a)	4. (d)	5. (c)	6. (a)	7. (b)
8. (b)	9. (c)	10. (d)	11. (b)	12. (d)	13. (c)	14. (c)
15. (b)	16. (d)	17. (b)	18. (b)	19. (a)	20. (c)	21. (d)
22. (a)	23. (a)	24. (c)	25. (c)	26. (a)	27. (a)	28. (b)
29. (d)	30. (c)	31. (d)	32. (c)	33. (b)	34. (d)	35. (b)
36. (c)	37. (a)	38. (a)	39. (a)	40. (d)	41. (a)	42. (d)
43. (b)	44. (a)	45. (c)	46. (c)	47. (a)	48. (d)	49. (d)
50. (a)	51. (c)	52. (b)	53. (c)	54. (c)	55. (a)	56. (c)
57. (a)	58. (a)	59. (a)	60. (b)	61. (b)	62. (d)	63. (a)
64. (c)	65. (b)	66. (b)	67. (c)	68. (c)	69. (c)	70. (c)
71. ( )	72. (c)	73. (a)	74. (a)	75. (c)	76. (b)	77. (a)
78. (d)	79. (a)	80. (b)	81. (c)	82. (d)	83. (c)	84. (a)
85. (c)	86. (d)	87. (a)	88. (b)			

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